

# NAVAL POSTGRADUATE SCHOOL

## Monterey, California



### **Removal of the Assumption of Cellular Targets in Computing Damage Aggregation to an Area Target from a Salvo of N Weapons**

by

I. Bert Russak  
Toke Jayachandran

September 1996 - December 1997

19980310 113

Approved for public release; distribution is unlimited.

**DTIC QUALITY INSPECTED 2**

Prepared for: Naval Postgraduate School  
Monterey, CA 93943

NAVAL POSTGRADUATE SCHOOL  
MONTEREY, CA 93943

Rear Admiral M. J. Evans  
Superintendent

R. Elster  
Provost

This report was prepared in conjunction with research conducted at the Naval Postgraduate School and funded by International Joint Warfare Analysis.

Reproduction of all or part of this report is authorized.

This report was prepared by:

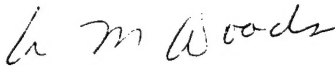


I. Bert Russak  
Associate Professor of Mathematics



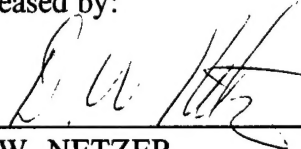
Toke Jayachandran  
Professor of Mathematics

Reviewed by:



WALTER M. WOODS  
Chairman, Department of  
Mathematics

Released by:



D. W. NETZER  
Associate Provost and Dean of  
Research

REPORT DOCUMENTATION PAGE			Form Approved OMB No. 0704-0188	
<small>Public reporting burden for this collection of information is estimated to average 1 hour per response, including the time for reviewing instructions, searching existing data sources, gathering and maintaining the data needed, and completing and reviewing the collection of information. Send comments regarding this burden estimate or any other aspect of this collection of information, including suggestions for reducing this burden, to Washington Headquarters Services, Directorate for Information Operations and Reports, 1215 Jefferson Davis Highway, Suite 1204, Arlington, VA 22202-4302, and to the Office of Management and Budget, Paperwork Reduction Project (0704-0188), Washington, DC 20503.</small>				
1. AGENCY USE ONLY (Leave blank)		2. REPORT DATE 17 December 1997		3. REPORT TYPE AND DATES COVERED Technical Report Sept 1996 - Dec 1997
4. TITLE AND SUBTITLE  Removal of the Assumption of Cellular Targets in Computing Damage Aggregation to an Area Target from a Salvo of n Weapons			5. FUNDING NUMBERS  DFR	
6. AUTHOR(S) I. Bert Russak and Toke Jayachandran				
7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)  Naval Postgraduate School Monterey, CA 93943-5000			8. PERFORMING ORGANIZATION REPORT NUMBER  NPS-MA-98-001	
9. SPONSORING/MONITORING AGENCY NAME(S) AND ADDRESS(ES)  International Joint Warfare Analysis Monterey, CA 93943-5000			10. SPONSORING/MONITORING AGENCY REPORT NUMBER	
11. SUPPLEMENTARY NOTES  The views expressed in the report are those of the authors and do not reflect the official policy or position of the Department of Defense of the United States of America.				
12a. DISTRIBUTION/AVAILABILITY STATEMENT  Approved for public release; distribution is unlimited.			12b. DISTRIBUTION CODE	
13. ABSTRACT (Maximum 200 words)  In the computation of damage aggregation to an area target from a salvo of N weapons, the assumption of a cellular target (i.e., a target composed entirely of cells) is often made. This assumption together with the implicit associated assumption of weapon strikes only at the center of those cells constitutes a significant simplification to the real-world model with potentially significant errors in the computation.  It is the purpose of this paper to eliminate the assumption of cellular targets.				
14. SUBJECT TERMS  estimation, weapons salvo, damage assessment, cellular target			15. NUMBER OF PAGES  10	
			16. PRICE CODE	
17. SECURITY CLASSIFICATION OF REPORT UNCLASSIFIED	18. SECURITY CLASSIFICATION OF THIS PAGE UNCLASSIFIED	19. SECURITY CLASSIFICATION OF ABSTRACT UNCLASSIFIED	20. LIMITATION OF ABSTRACT	

# **Removal of the Assumption of Cellular Targets in Computing Damage Aggregation to an Area Target from a Salvo of $n$ Weapons**

by

**Associate Professor I. Bert Russak  
Professor Toke Jayachandran**

**Department of Mathematics  
Naval Postgraduate School  
Monterey, CA 93943**

**December 17, 1997**

## **ABSTRACT**

In the computation of damage aggregation to an area target from a salvo of  $N$  weapons, the assumption of a cellular target (i.e., a target composed entirely of cells) is often made. This assumption together with the implicit associated assumption of weapon strikes only at the center of those cells constitutes a significant simplification to the real-world model with potentially significant errors in the computation.

It is the purpose of this paper to eliminate the assumption of cellular targets.

## Background

The analysis of the damage aggregation to an area target resulting from a salvo of  $n$  weapons fired at it, is a difficult topic. Under the assumption that each target point is equally likely to be hit and using a simple "cookie cutter" area of destruction (kill area) from a single hit, then this difficulty results in large part from the great geometric variety of the total damaged area that results from  $k$  ( $k \leq n$ ) hits. The assumption of a cellular target, i.e. a target composed entirely of cells with predetermined locations greatly simplifies the problem. If it is further assumed that a hit **anywhere in a particular cell destroys that cell and no other cells** even if that hit occurs near the boundary of adjacent cells, then determining the geometry of the damaged area becomes more tractable.

For example, consider the target of figure 1 consisting of twelve cells. Then a hit, indicated by "X" near the boundary of the shaded cell would in the basic model of [1] and [2] damage the shaded cell and no other cell. Effectively this assumes that hits only occur at the centers of the predetermined cells.

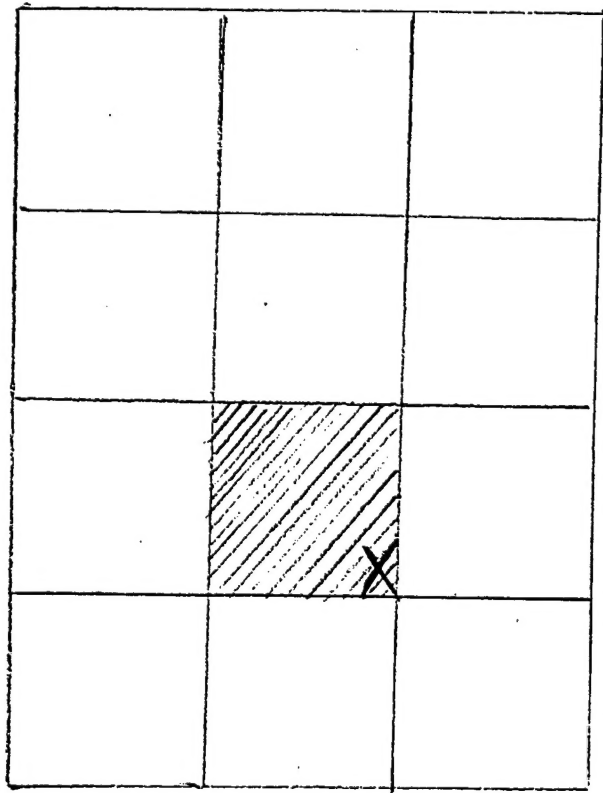


Figure 1

However since hits do not in reality always occur in this manner, then such a model does little to represent the real world situation.

It is the purpose of this work to remove the assumption of predetermined cell location and to thus allow hits to occur anywhere in the target. In order to do this we will first use predetermined location cells, forming in essence a partition of the target. We will then form a sequence of such partitions of smaller and smaller cells where in the limit, these cells reduce to points.

For example, using a target of size 100 square feet with square cells each having a cell size of  $(2/3)^2$  square feet, then the partition will be as shown in Figure 2.

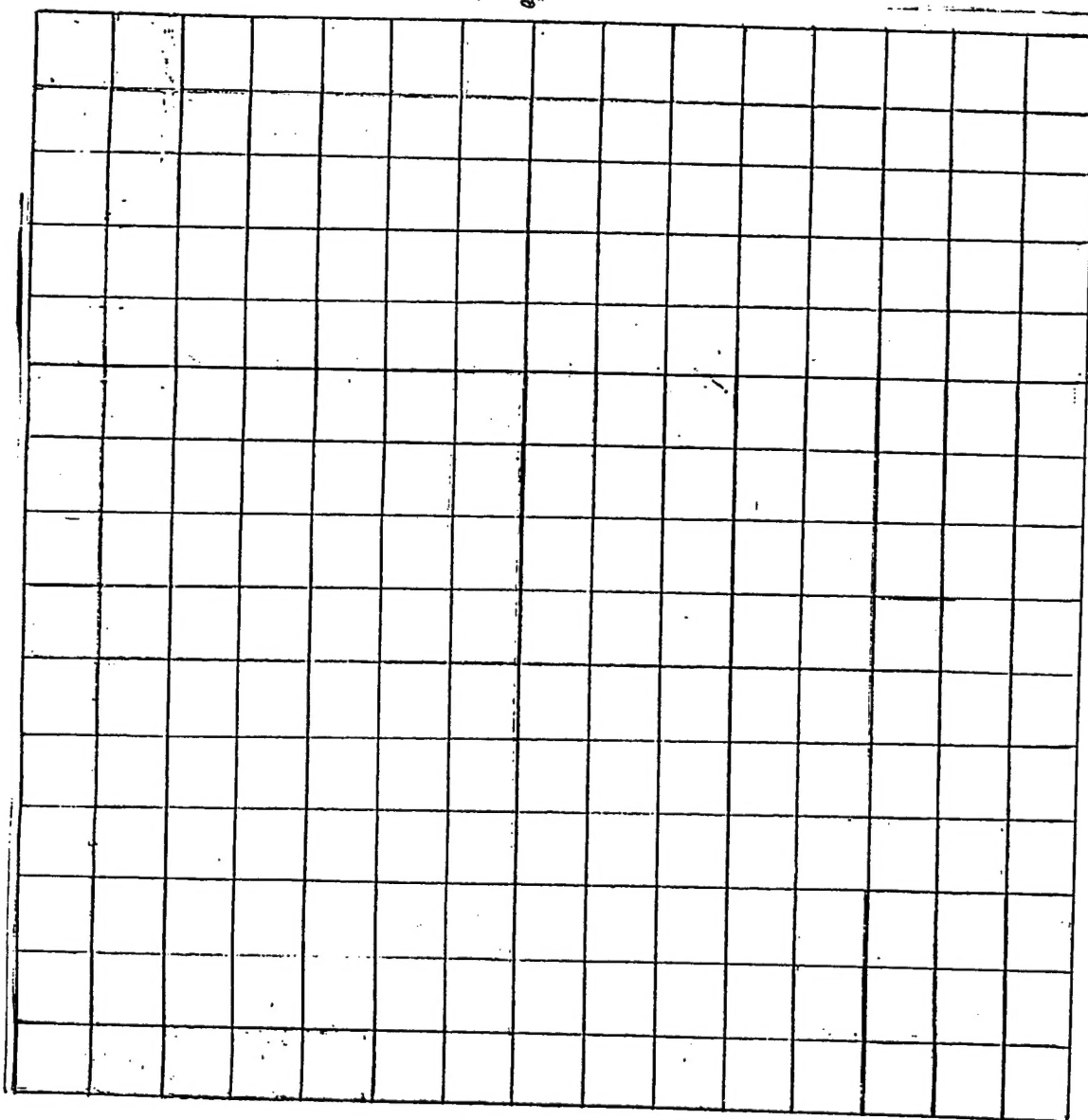
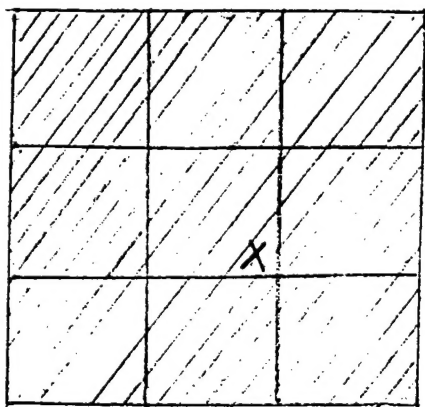


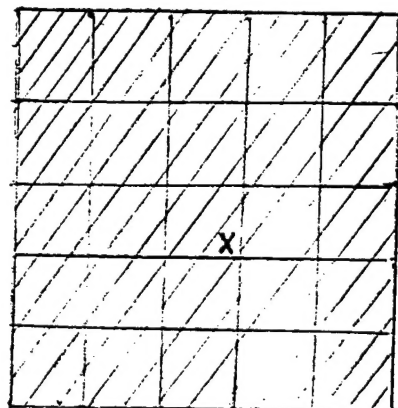
Figure 2

In the proposed limiting process, as the cells reduce in size, the above model error (in essentially assuming that a hit anywhere in a cell is a hit in the center of that cell) becomes less and less significant. Of course as the cell size reduction occurs, the kill area (which is a constant) will be composed of more than one cell and in fact an increasing number of cells. Thus for any fixed partition of the target, a hit to a cell will in general damage that cell in addition to a number of surrounding cells whose total area equals the kill area<sup>[1]</sup>. The actual cell that was hit will then be at the center of this kill area. For example, continuing with square cells and assuming a square kill area of 4 square feet and using two different partitions with cells of size  $(2/3)^2$ ,  $(2/5)^2$  square feet respectively, then the pictures are (not to scale)



*kill area=4 square feet  
cell size= $(2/3)^2$  square feet*

Figure 3



*kill area=4 square feet  
cell size= $(2/5)^2$  square feet*

Figure 4

Clearly, the error between the area actually damaged (which would be the kill area centered at the "X" ... this is not displayed) and the area damaged according to the model (which is the kill area centered at the center of the hit cell...this is displayed) is smaller in Figure 4 (with smaller cells) than in Figure 3<sup>[2]</sup>. In the limit, as the cell size becomes smaller and smaller (with limit being a point) then the above noted error should reduce to a limit of zero,

<sup>[1]</sup> The partitions are chosen so that an integral number of cells make up the kill area.

<sup>[2]</sup> The above assumes that the hit cell is sufficiently interior (i.e. within the target boundary) to the target so as to have a kill area (of which it is the center) entirely contained in the target. If this is not the case for a hit cell then appropriate modifications (to be explained later) are made.



## Explanation of Solution Technique

We continue with the explanation of our partitions. For simplicity of presentation we assume a square target of side  $L$  and a weapon with a square kill area (cookie cutter area of destruction) of side  $2R$ . We use  $n=1,2,3,\dots$  as the index of our partitions. That is each value of  $n$  will represent a partition of the target. For any  $n$  our cell size will be  $(2R/(2n+1))^2$  square feet.

The number of *interior* target cells along a side of the target is then  $L/(2R/(2n+1))^{[3]}$  and then the total number of interior target cells is<sup>[4]</sup>  $m_n = (L/(2R/(2n+1)))^2$ . In our model we shall consider a hit to the target as a hit to an interior cell or to an "exterior" cell for we shall allow  $n$  rows and  $n$  columns of exterior cells (which surround the target) in the partition represented by  $n$  (see Figures 5 and 6 for the cases  $n=1$  and  $n=2$ ). Counting the interior and exterior cells, then there is a total of  $L/(2R/(2n+1)) + 2n$  cells along a side of the target. This results in a grand total of  $T_n = (L/(2R/(2n+1)) + 2n)^2$  interior and exterior cells.

Now for a fixed value of  $n$ , we let

$$(1) \quad C_i(k) = \begin{cases} 1 & \text{if the } i^{\text{th}} \text{ cell is damaged after } k \text{ hits} \\ 0 & \text{if the } i^{\text{th}} \text{ cell is not damaged after } k \text{ hits} \end{cases}$$

Next, we have the following: according to our defined kill area and cell size, then, for an undamaged target, a hit to a cell will damage a square of  $(2R/(2R/(2n+1)))^2 = (2n+1)^2$  cells around (and including) the cell that was hit. We call this the "kill area around the cell." Also, if one examines Figures 5 and 6, it will be clear that any interior cell  $\bar{C}$  may be damaged either by a direct hit or by a hit to any other cell in the kill area<sup>[5]</sup> around  $\bar{C}$ . Thus any interior cell can be damaged in  $(2n+1)^2$  ways (all equally likely). Since there are a total of  $T_n$  (equally likely) cells to hit, then for the  $n^{\text{th}}$  cell partition

$$(2) \quad \begin{aligned} P_n[C_i(1)=1] &= (2n+1)^2 / T_n = ((2n+1)/(L/(2R/(2n+1)) + 2n))^2 \\ &= (((2n+1)2R)/(L(2n+1) + 4nR))^2 \end{aligned}$$

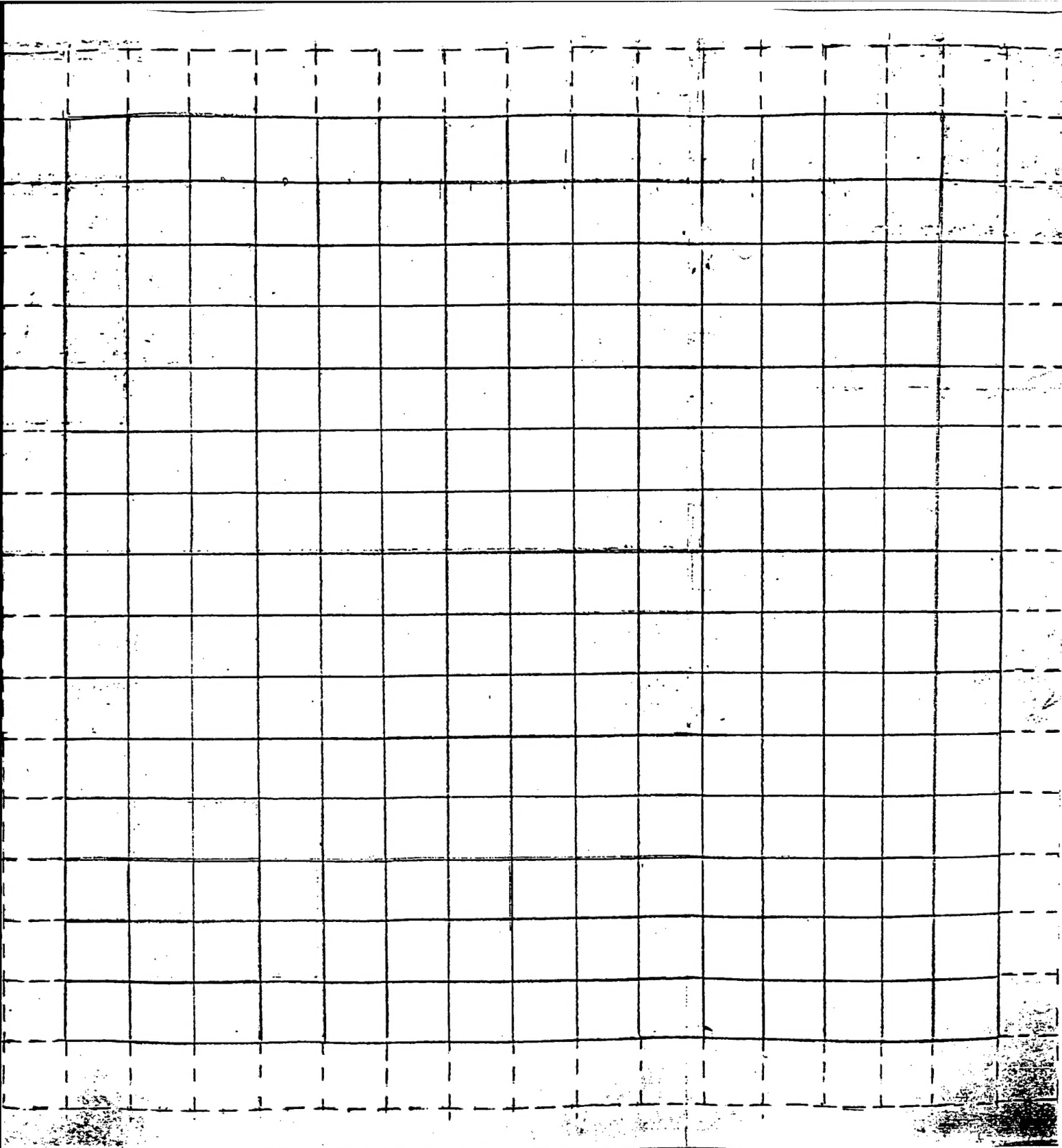
where  $P_n[C_i(1)=1]$  refers to the probability, in the  $n^{\text{th}}$  partition of the target, of the  $i^{\text{th}}$  cell being damaged after the first hit.

We now consider points (as well as interior or exterior cells) in the target. Then for the point with coordinates  $(x,y)$ , let

<sup>[3]</sup> We shall assume that  $L/2R$  is an integer.

<sup>[4]</sup> We number our cells so that the first  $m_n$  cells are the interior cells.

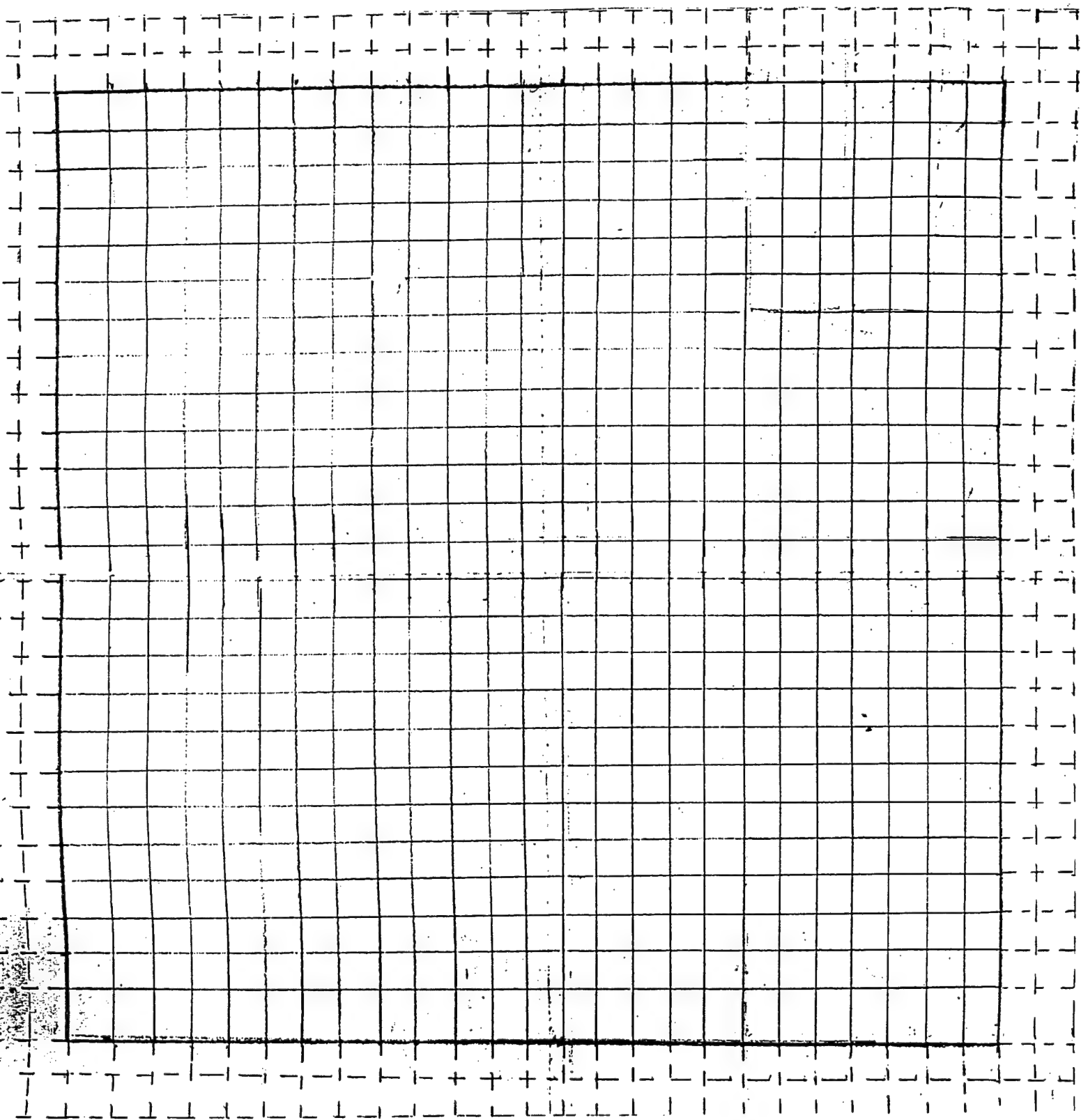
<sup>[5]</sup> Where such kill area may include exterior cells (in the indicated  $n$  rows and  $n$  columns outside the target) as shown in Figures 5 and 6.



*Exterior cell*

*Interior cell*

Figure 5  
Target Partitioning for  $n=1$



*Exterior cell*

*Interior cell*

Figure 6  
Target Partitioning for  $n=2$

$$(3) \quad (x,y)(k) = \begin{cases} 1 & \text{if the point } (x,y) \text{ is damaged after } k \text{ hits} \\ 0 & \text{if the point } (x,y) \text{ is not damaged after } k \text{ hits} \end{cases}$$

Now for the  $n^{\text{th}}$  partition, the arbitrary point  $(x,y)$  is in some cell (perhaps more than one cell if it is on the boundary of a cell, in which case use any one of those cells)  $C_{i_n}$ . Thus

$$(4) \quad P_n[(x,y)(1)=1] = P_n[C_{i_n}(1)=1] = ((2n+1)^2)/T_n$$

where  $P_n[(x,y)(1)=1]$  is the probability (in the  $n^{\text{th}}$  partition) of  $(x,y)$  being damaged after the first hit to the target. Taking the limit as  $n \rightarrow \infty$  to get the true value of the probability of  $(x,y)$  being damaged after the first hit, i.e.  $P[(x,y)(1)=1]$ , we have

$$(5) \quad \begin{aligned} P[(x,y)(1)=1] &= \lim_{n \rightarrow \infty} P_n[C_{i_n}(1)=1] = \lim_{n \rightarrow \infty} ((2n+1)2R/(L(2n+1) + 4nR))^2 \\ &= \left( \lim_{n \rightarrow \infty} (2n+1)2R/(L(2n+1) + 4nR) \right)^2 \\ &= \left( \lim_{n \rightarrow \infty} (2 + 1/n)2R/(L(2 + 1/n) + 4nR) \right)^2 = (4R/(2L + 4R))^2 \end{aligned}$$

Now let  $\Delta(k)$  be the portion of the target damaged after  $k$  hits. Also, let

$$w(x,y) = \begin{cases} 1/L^2 & \text{if } (x,y) \text{ is interior point (uniform weighting)} \\ 0 & \text{if } (x,y) \text{ is exterior point} \end{cases}$$

Then for  $k=1$

$$(6) \quad \Delta(1) = \iint_{\text{Target}} w(x,y)(x,y)(1) dy dx$$

and with  $D(k)$  as the expected value of  $\Delta(k)$ , then again for  $k=1$

$$(7) \quad D(1) = E[\Delta(1)] = E \left[ \iint_{\text{Target}} w(x,y)(x,y)(1) dy dx \right] = \iint_{\text{Target}} w(x,y) E[(x,y)(1)] dy dx$$

with

$$(8) \quad E[(x,y)(1)] = 1P[(x,y)(1)=1] + 0P[(x,y)(1)=0] = P[(x,y)(1)=1] = \delta_{x,y}(1)$$

where  $\delta_{x,y}(1)$  is shorthand for  $P[(x,y)(1)=1]$ . Since we assume that all points in our target are equally likely to be hit, then we drop the dependence upon  $(x,y)$  and write  $\delta(1)$ . Also by the above listed values for  $w(x,y)$  we have

$$(9) \quad D(1) = \iint_{\text{TargetInterior}} 1/L^2 \delta(1) dydx = \delta(1)/(L^2)L^2 = \delta(1) = (4R/(2L + 4R))^2.$$

This is the expected value of the portion of the target damaged after the first hit ( $k=1$ ). Continuing on for general  $k$ :

$$(10) \quad P[(x, y)(1) = 0] = 1 - P[(x, y)(1) = 1]$$

and assuming independent hits

$$(11) \quad P[(x, y)(k) = 0] = (P[(x, y)(1) = 0])^k = (1 - P[(x, y)(1) = 1])^k$$

so that by (5)

$$(12) \quad P[(x, y)(k) = 1] = 1 - P[(x, y)(k) = 0] = 1 - (1 - P[(x, y)(1) = 1])^k = 1 - (1 - (4R/(2L + 4R))^2)^k.$$

Then analogous to (6):

$$(13) \quad \Delta(k) = \iint_{\text{Target}} w(x, y)(x, y)(k) dydx$$

and analogous to (8) and using (12):

$$(14) \quad \begin{aligned} E[(x, y)(k)] &= 1P[(x, y)(k) = 1] + 0P[(x, y)(k) = 0] \\ &= P[(x, y)(k) = 1] = 1 - (1 - (4R/(2L + 4R))^2)^k \end{aligned}$$

so finally

$$(15) \quad \begin{aligned} D(k) &= E[\Delta(k)] = E\left[\iint_{\text{Target}} w(x, y)(x, y)(k) dydx\right] = \iint_{\text{Target}} w(x, y)E[(x, y)(k)] dydx \\ &= \iint_{\text{TargetInterior}} \frac{1}{L^2} [1 - (1 - (4R/(2L + 4R))^2)^k] dydx = 1 - [1 - (4R/(2L + 4R))^2]^k \end{aligned}$$

This represents the expected portion of the target damaged after  $k$  hits. As a comparison exercise, we select  $L=10$  and  $R=1$  as representative numbers for our parameters and compute  $D(1)$ :

$$(16) \quad D(1) = 1 - [1 - (4/(20 + 4))^2] = .0277778.$$

In order to compute the corresponding values of  $D$  for some partitions, we define  $\Delta_n(1)$ ,  $D_n(1)$  respectively, as the portion of the target damaged after the first hit and its expected value both

computed in the  $n^{\text{th}}$  partition of the target. We also define  ${}_nw_i$  as the weighting given to the  $i^{\text{th}}$  cell in the  $n^{\text{th}}$  partition:

$${}_nw_i(C_i) = \begin{cases} \frac{1}{m_n} & \text{if } C_i \text{ is interior cell} \\ 0 & \text{if } C_i \text{ is exterior cell} \end{cases}$$

Then working with cells (rather than points) in the  $n^{\text{th}}$  partition and following a parallel development to 6)→9) gives by (4) and (2)

$$(17) \quad \Delta_n(1) = \sum_{i=1}^{T_n} {}_nw_i(C_i) C_i(1) = \sum_{i=1}^{m_n} \frac{1}{m_n} C_i(1)$$

$$(18) \quad D_n(1) = E[\Delta_n(1)] = E\left[\sum_{i=1}^{m_n} \frac{1}{m_n} C_i(1)\right] = \sum_{i=1}^{m_n} \frac{1}{m_n} E[C_i(1)]$$

where

$$(19) \quad E[C_i(1)] = 1P_n[C_i(1)=1] + 0P_n[C_i(1)=0] = P_n[C_i(1)=1] = P_n[C(1)=1] = \frac{(2n+1)^2}{T_n}$$

where the next to last equality results from the equal likeliness of all cells (in the  $n^{\text{th}}$  partition) to be damaged after the first hit (so the dependency on  $i$  is removed). So then by (2):

$$(20) \quad D_n(1) = \sum_{i=1}^{m_n} \frac{1}{m_n} \frac{(2n+1)^2}{T_n} = \frac{(2n+1)^2}{T_n} = \left( \frac{(2n+1)2R}{L(2n+1) + 4nR} \right)^2.$$

Computing  $D_n(1)$  for some values of  $n$ :

$$\text{For } n=1, D_1(1) = (6/34)^2 = .031142$$

$$\text{For } n=2, D_2(1) = \left( \frac{5(2)}{10(5)+8} \right)^2 = (10/58)^2 = .029727$$

$$\text{For } n=3, D_3(1) = \left( \frac{7(2)}{10(7)+12} \right)^2 = (14/82)^2 = .029149.$$

This is a decreasing sequence with limit .0277778 which represents the expected damage after one hit without the assumption of predetermined cell locations.

As noted above, the removal of predetermined cell location is a significant step towards making a more real-world model for damage aggregation.

Also, this is a first step in using the concept of a limiting process to delete the predetermined cellular construction of the target. Certain construction techniques used herein such as square targets, seem to be removable in generalizing this approach. For example, it seems that any target which is decomposable into a finite number of square sub-targets might be solvable by this approach. This is certainly an area for future work.

### **Bibliography**

- [1] Esary, J.D., Studies on Damage Aggregation for Weapons Salvos. NPS Technical Report NPS-55-90-16
- [1] Esary, J.D., Studies on Damage Aggregation for Weapons Salvos (2). NPS Technical Report NPSOR 92-007

## DISTRIBUTION LIST

	No. Of copies
Defense Technical Information Center 8725 John J. Kingman Rd., STE 0944 Ft. Belvoir, VA 22060-6218	1
Dean of Research Naval Postgraduate School, Code 09 Monterey, CA 93943-5000	1
Library Code 013 Naval Postgraduate School Monterey, CA 93943-5100	2
Chairman, Code MA Department of Mathematics Naval Postgraduate School Monterey, CA 93943	2
Professor Gordon Schacher Director of IJWA c/o Code JW/Wm Naval Postgraduate School Monterey, CA 93943-5000	2
Professor Toke Jayachandran Department of Mathematics Naval Postgraduate School Monterey, CA 93943-5000	1
Professor I. Bert Russak Department of Mathematics Naval Postgraduate School Monterey, CA 93943-5000	2
CAPT George Conner Department of Operations Research Naval Postgraduate School Monterey, CA 93943	2
Lecturer Paul Bloch Deputy Director Of IJWA Naval Postgraduate School Monterey, CA 93943	2